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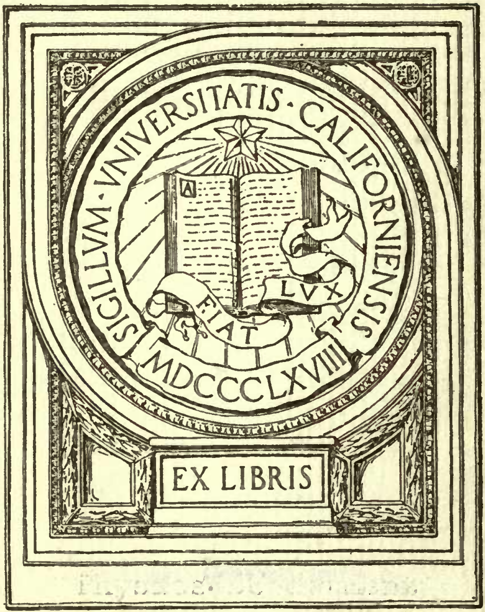

LOWER DIVISION

A SUPPLEMENT

TO

NICHOLS AND FRANKLIN'S

Electricity and Magnetism

A PAMPHLET

SUPPLEMENTARY TO THE SECOND VOLUME

OF

Nichols and Franklin's Elements of Physics

CONTAINING A REVISION OF THE CHAPTERS

ON

Electrostatics and Self-induction

TOGETHER WITH SUGGESTIONS

AS TO THE

ARRANGEMENT OF LESSONS

IN THE USE OF THE VOLUME AS A TEXT BOOK

UNIVERSITY OF CALIFORNIA
DEPARTMENT OF PHYSICS

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~~PHYSICS DEPT.~~

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PREFACE.

THE main object of this pamphlet is to place in the student's hands a revision of certain chapters in the second volume of Nichols and Franklin's Elements of Physics, namely, the Chapters on Electrostatic Theory and the Chapter on Self-Induction. A few changes of less extent are also indicated and an *Outline of Lessons* is suggested in order that the scattered material included in the Text-Book and in this pamphlet may be easily available in class-room work.

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OUTLINE OF LESSONS.

LESSON I. Magnetism and the magnetic field.

Articles 327–345.

LESSON II. Gauss's method for measuring magnetic field and magnetic moment. Magnetic lines of force. Magnetic figures. The magnetism of iron.

Articles 346, 348, 349 and 510–512.

A correction to Article 346 is given in this pamphlet, Art. 1.

LESSON III. The electric current. Production of magnetic field by the electric current. The tangent galvanometer.

Articles 352–362, omitting 357 and 361. Also Article 2 of this pamphlet instead of Article 358.

LESSON IV. Action of a magnetic field on a coil. The electrodynamometer. Sensitive galvanometers. The D'Arsonval galvanometer.

Article 3 of this pamphlet and Articles 364–366.

LESSON V. Joule's law and electrical resistance. Ohm's law and electromotive force.

Articles 367–378 and Article 4 of this pamphlet.

LESSON VI. Shunts.

Articles 379–382.

LESSON VII. Magnetic flux. Induced e. m. f. The dynamo.

Articles 5 and 6 of this pamphlet and Articles 533–536.

LESSON VIII. Electrolysis and batteries.

Articles 391–408.

Articles 395–398 may be omitted.

LESSON IX. Electrical measurements.

Articles 409–430 and Articles 7–11 of this pamphlet.

LESSONS X., XI. and XII. Electric charge. Electrostatics.

These lessons are to be found in this pamphlet, Articles 19–55.

Lesson XIII. The phenomena of discharge.
Articles 484–502.

Lessons XIV., XV. and XVI. Magnetism of iron.
Introduction, Articles 503–509 or Articles 12–15 of this pamphlet.
Magnetism of iron, Articles 513–532.

Lessons XVII. and XVIII. Inductance.
Articles 56–68 of this pamphlet.

Lesson XIX. Thermo-electricity. Practical applications of electricity.
Articles 558–580.
Substitute Article 16, this pamphlet, for Article 575.

Lesson XX. Mechanical conceptions of electricity and magnetism.
Articles 594–603 and Articles 17, 18 of this pamphlet.

1. Correction to article 346.—After equation (iii), page 24, omit to page 25, "The uncertain quantity....," and substitute the following:

This equation may be simplified as follows:

Reduce the fractions

$$\frac{m}{\left(d-\frac{l}{2}\right)^2} \quad \text{and} \quad \frac{m}{\left(d+\frac{l}{2}\right)^2}$$

to a common denominator and substitute M for ml in accordance with equation (191) and we have:

$$h = 2Md\frac{1}{\left(d^2-\frac{l^2}{4}\right)^2}.$$

Multiply numerator and denominator of the second member of this equation by

$$\left(d^2+\frac{l^2}{4}\right)^2$$

and we have:

$$h = 2Md\frac{d^4+\frac{d^2l^2}{2}+\frac{l^4}{16}}{\left(d^4-\frac{l^4}{16}\right)^2}.$$

In this expression $\frac{l^4}{16}$ may be dropped, being negligible. Therefore

$$h = \frac{2M}{d^5}\left(d^2 + \frac{l^2}{2}\right). \tag{iv}$$

Substitute this simplified value of h in the expression for $\tan\varphi\left(=\frac{h}{H}\right)$ and we have:

$$\tan\varphi = \frac{2M}{Hd^5}\left(d^2 + \frac{l^2}{2}\right). \tag{v}$$

The large magnet may now be placed near to NS, say at distance d_1, and the corresponding angle of deflection being φ_1, we have in a similar manner:

$$\tan\varphi_1 = \frac{2M}{Hd_1^{\,5}}\left(d_1^{\,2} + \frac{l^2}{2}\right) \tag{vi}$$

2. Magnetic field intensity at the center of a circular coil.—Consider a magnetic pole of strength m placed at the center of a circular coil of radius r. Let I be the current in the coil and n the number of turns of wire. The intensity of magnetic field at the wire due to the pole m is $\frac{m}{r^2}$ according to equation (193). This field is everywhere at right angles to the wire as shown by the lines of force in Fig. 1. The length of wire in the coil is $2\pi rn$.

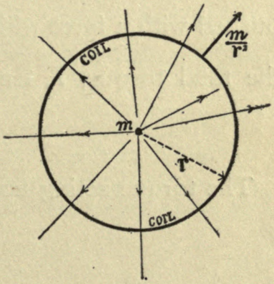

FIG. 1.

Therefore the force which pushes the wire sidewise is $I.2\pi nr\cdot\frac{m}{r^2}$ according to equation (199). An equal and opposite force is exerted upon the pole by the current in the wire; but this force $I.2\pi nr\cdot\frac{m}{r^2}$ which acts upon the pole is, according to equation (192), equal to the product of the strength of the pole into the intensity of the magnetic field f at the pole due to the current. That is $I.2\pi nr\frac{m}{r^2} = mf.$ Therefore

$$f = \frac{2\pi nI}{r}. \qquad (204)$$

3. Action of a magnetic field upon a rectangular coil.—Consider a rectangular coil of wire cc, Fig. 2, suspended, so as to be free to rotate about the axis pp, in a uniform magnetic field of intensity f as shown. Let b be the breadth of the coil, a its length, n the number of turns of wire in the coil, and I the current flowing in the coil. The wires ww in one side of the coil have a total length an and are pushed sidewise by the field with a force $I.\,an.\,f$ according to equation (199). The torque action of this force about the axis pp is $\frac{b}{2}.I.an.f.$ The wires which form the other side of the coil are pushed with a force which also produces a torque $\frac{b}{2}.I.an.f$ so that the total torque, T, tending to turn the coil about the axis pp is

$$T = abnIf.$$

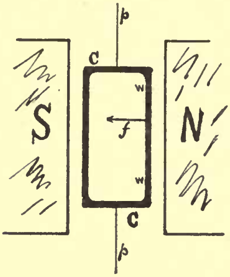

FIG. 2.

The torque acting upon a circular coil of radius r is

$$T = \pi r^2 nIf. \qquad (210)$$

4. Conductors and insulators.—A substance through which the electric current can pass easily is called a *conductor*. The metals, carbon, and salt solutions are good conductors. A substance through which the electric current can scarcely pass at all is called an *insulator*. Air, glass, and hard rubber are good insulators. By using a very sensitive galvanometer it may be shown that a perceptible current may be made to pass through almost any insulator. A wire, or any conductor, is said to be *insulated* when it is separated by insulating substances from all neighboring conductors. For example, telegraph and telephone wires are supported on knobs of glass or porcelain, called *insulators.* Wire for use in electrical instruments is insulated by a covering of cotton or silk.

5. Magnetic flux.—Let a be an area at right angles to the velocity of a moving fluid, and let v be the velocity of the fluid. Then av is the flux of fluid across the area in units volume per second. Similarly the product of the intensity, f, of a magnetic field into an area a at right angles to f is called the *magnetic flux* across the area. That is

$$N = fa \tag{i}$$

in which N is the magnetic flux across an area a which is at right angles to a magnetic field of intensity f.

The unit of magnetic flux is the flux across one square centimeter of area at right angles to a magnetic field of unit intensity. This unit flux is called a *line of force** or simply a *line.* For example, the intensity of the magnetic field in the air gap between the pole face of a dynamo and the armature core is, say, 5,000 units, and this field is normal to the pole face of which the area is 300 square centimeters, so that 1,500,000 lines of magnetic flux pass from the pole face into the armature core.

The magnetic flux across an extended surface in a magnetic field cannot be expressed as the product of the field intensity into the area of the surface as in equation (i) for the reason that the field will in general vary in intensity from point to point and also for the reason that the field may not be at right angles to the surface at every point. In this case we have

$$N = \Sigma f \cos \varepsilon . \triangle S \tag{ii}$$

in which $\triangle S$ is an element of the surface, f is the field intensity at the element, $f \cos \varepsilon$ is the resolved part of f perpendicular to $\triangle S$, and $f \cos \varepsilon . \triangle S$ is the flux across the element.

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* A line of force is a line drawn in a magnetic field so as to be in the direction of the field at each point. The term *line of force* is used for the unit flux for the following reason: Consider a magnetic field. Imagine a surface drawn across this field. Suppose this surface to be divided into *parts* across each of which there is unit flux. Imagine lines of force drawn in the magnetic field so that one line of force passes through each of the *parts* of our surface. Then the magnetic flux across any area anywhere in the field will be equal to the *number of these lines* which cross the area.

6. Magnetic flux from a magnetic pole of strength m.—*Proposition.* The number of lines of force which emanate from a magnetic pole of strength m is

$$N = 4\pi m. \tag{iii}$$

Proof: Imagine a spherical surface of radius r drawn with the pole m at its center. The area of this spherical surface is $4\pi r^2$, the field intensity at this surface due to the pole is $\frac{m}{r^2}$ according to equation (193), and this field is everywhere at right angles to the spherical surface. Therefore the magnetic flux across the surface is $\frac{m}{r^2} \cdot 4\pi r^2 = 4\pi m$ according to equation (i).

7. Substitute for Fig. 218, page 93.

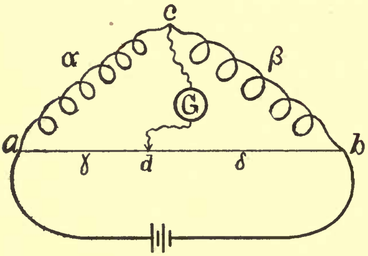

FIG. 218.

8. The measurement of resistance by the ammeter and voltmeter.—In the Testing Laboratory, where it is often inconvenient to use Wheatstone's bridge, resistance is most frequently measured by means of an ammeter and a voltmeter. A current measured by an ammeter is sent through the resistance to be measured and the e. m. f. between the terminals of the resistance is measured by a voltmeter. The resistance is then calculated from equation (217).

9. Measurement of very high resistances. *Insulation resistance.*—Very large resistances, for example, the resistance of the covering of an insulated wire, cannot be easily measured by the methods adapted to the measurement of the resistances of wires. These very high resistances may be determined by measuring with a very sensitive galvanometer the current I which is forced through the given resistance by a large e. m. f., E, which is known.

The resistance is then equal to $\frac{E}{I}$ by equation (217).* For example, the resistance of the insulation of a lead-encased cable was found by this method to be 5,000 megohms (one megohm is 1,000,000 ohms).

11. The electrical measurement of power.—(*a*) The power expended in an electrical circuit may be calculated from the equation $P = EI$ (219) when the current, I, in the circuit and the e. m. f., E, between the terminals of the circuit have been measured. It is in this way that the power expended in electrical circuits is most frequently measured.

(*b*) The power expended in heating a circuit may be calculated from the equation $P = RI^2$ (216), when the resistance R of the circuit is known and the current I in the circuit has been measured.

Example. An electric motor of which the resistance 0.24 ohms takes 25 ampères of current from electric mains between which the e. m. f. is 110 volts. The total power EI taken from the mains is 2,750 watts, and the power RI^2 expended in heating the wires of the motor is 150 watts.

(*c*) The power expended in a circuit may be measured by means of the *wattmeter*.

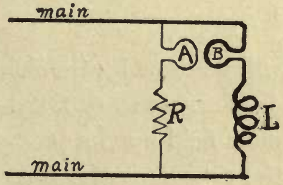

FIG. 3.

11. The Wattmeter is an electro-dynamometer the movable coil of which, A, Fig. 3, carries a pointer which plays over a divided scale. This coil is made of fine wire and, together with a resistance R, is connected directly to the mains. The other coil B of the electro-dynamometer is connected in series with the circuit L in which the power to be measured is expended. The current in the coil A is $\frac{E}{R}$. The current in the coil B is the current I which flows through the circuit L. The force with which the coil B tends to deflect the coil A is proportional to the product $\frac{E}{R} \cdot I$ of the currents in the two coils. That is, this force, since R is constant, is proportional to the power EI expended in the circuit L. Therefore, the scale of the instrument may be divided to read watts directly.

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*As a matter of fact, insulators do not conform to Ohm's law or, in other words, the current through an insulator is not strictly proportional to the e. m. f., so that different values will be gotten for the insulation resistance according to the value of e. m. f. used.

12. Intensity of magnetic field in a long coil.—The intensity f of the magnetic field in a long coil of wire is

$$f = 4\pi ni \tag{276a)*}$$

in which n is the turns of wire per centimeter length of coil and i is the current in the coil. If i is expressed in ampères then

$$f = \frac{4\pi}{10} \cdot ni. \tag{276b}$$

13. Work done on a moving magnetic pole. Magnetomotive force.—Consider a uniform magnetic field of intensity f. A magnetic pole of strength m placed in this field is pulled by a force mf according to equation (192). If this pole is allowed to move a distance l in the direction of this force an amount of work

$$W = mlf \tag{i}$$

will be done by the field on the pole.

Magnetomotive force.—The *work per unit pole* done by a magnetic field upon a pole as it is carried over a given path is called the *magnetomotive force* (m. m. f.) along the path. That is

$$\text{m. m. f.} = \frac{W}{m} \tag{ii}$$

in which W is the work done by a magnetic field upon a pole as it is carried along a given path and m. m. f. is the magnetomotive force along the path. From equations (i) and (ii) we have

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* The proof of this equation is given on page 174 of the text-book.

$$\text{m. m. f.} = lf. \tag{iii}$$

That is the m. m. f. along a path of length l parallel to a magnetic field of uniform intensity f is lf.

14. General relation between magnetic field and m. m. f.—Consider an element $\triangle s$ of a path pp', Fig. 4, in a magnetic field. Let f be the intensity of the field at the element. Then mf is the force which would act on a pole m at the element, $mf. \cos \varepsilon$ is the resolved part of this force parallel to $\triangle s$, and $mf \cos \varepsilon \,.\, \triangle s$ is the work done on the pole by this force as the pole is moved along the element. The total work done on the pole as it is carried from p to p' is $W = \Sigma mf \cos \varepsilon \,.\, \triangle s$ or

$$W = m\Sigma f \cos \varepsilon \,.\, \triangle s.$$

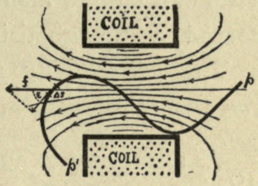

FIG. 4.

Therefore the m. m. f. $\frac{W}{m}$, along the path pp' is

$$\text{m. m. f.} = \Sigma f \cos \varepsilon. \triangle s \tag{iv}$$

15. Magnetomotive force of a coil.—The m. m. f. along a path which passes once around a wire carrying a current i is

$$\text{m. m. f.} = 4\pi i. \tag{v}^*$$

The m. m. f. along a path, such as the dotted lines in Figs. 5a and 5b, which passes around a bundle of Z wires through each of which a current i is flowing is

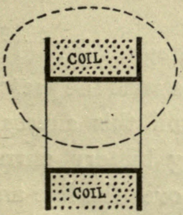

FIG. 5a.

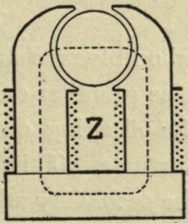

FIG. 5b.

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* The proof of this equation is given on page 173 of the text-book.

$$\text{m. m. f.} = 4\pi Zi. \qquad \text{(vi)}$$

If the current is expressed in ampères this equation becomes

$$\text{m. m. f.} = \frac{4\pi}{10} Zi. \qquad \text{(vii)}$$

As used in dynamo electric machinery, the electromagnet consists of a circuit, mainly of iron, through which magnetic flux is forced by a magnetizing coil or coils Z, Fig. 5b.

The *magnetizing power* of these coils is the *magnetomotive force* they produce around the iron circuit and this is equal to $\frac{4\pi}{10} Zi$ according to equation (vii) in which Z is the total number of turns of wire in the coils and i is the current in ampères which flows through the coils.

16. The Carbon Transmitter.—The current from a battery passes through the primary of a small induction coil and through a mass of granular carbon *cc*, Fig. 6. This carbon lies loosely between a carbon block B and a diaphragm DD. The electrical resistance of the granular carbon varies with the pressure exerted upon it by the vibrating diaphragm and this causes the battery current to fluctuate. This fluctuating battery current induces currents in the secondary which pass over the line and effect the telephone receiver at the distant station.

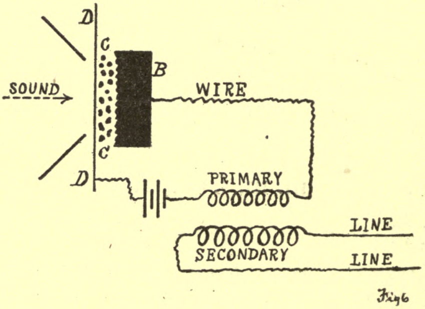

FIG. 6.

17. The conception of the production of e. m. f when a conductor moves across a magnetic field.—The e. m. f. induced in a moving body shows itself as an electric field the lines of force of which emanate from the ends of the moving conductor. Consider a conductor *A* (Fig. 7), which is moving at velocity *v* across a magnetic field of intensity *f* perpendicular to the paper and towards the reader. This body being a conduct or, the ether inside of it cannot sustain electric stress, so that the energy of the magnetic field, which is seated in the region immediately in front of *A*, cannot be geared across *A*, but an electric field is called into existence about the ends of *A* as indicated by the lines of force, and the magnetic energy is geared around *A* as indicated by the lines marked *energy stream*.

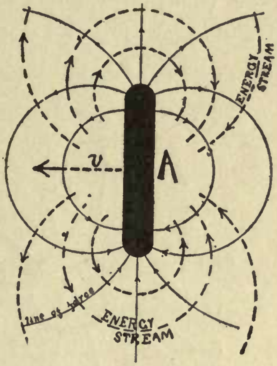

FIG. 7.

18. Mechanical conception of the Hertz oscillator.—The Hertz oscillator or vibrator is described on page 165 of the text-book and depicted in Fig. 288.

The action of the Hertz vibrator is as follows :

Let *AB* (Fig. 8) be the balls of the vibrator upon which electric charge has been collecting. Consider a chain of cells which, when undistorted, lies along the dotted line, everywhere perpendicular to the lines of force. When *A* is positively charged, this chain is distorted as shown (in part), but since it is a closed chain, this distortion is fixed. When a spark occurs at the gap, a line of slip is established across this chain, and the distortion disappears as explained in Art. 602.

If the slip takes place with great friction (high electrical resistance in the gap), the cells at the spark begin turning slowly, and the entire energy of the electric field is geared into the gap and changed to heat. If the slip is almost frictionless (low electrical resistance), the electrical energy is used mostly in overcoming the inertia of the cells as they are set rotating, and after a very short interval of time a very large part of the electrical energy will have been converted into kinetic energy of the rotating cells (magnetic energy). During this conversion the energy, streaming along the dotted line, largely disappears from the regions *ee*, and is distributed mainly in the region *mm*. When the chain of cells has been freed from distortion, the rotary motion of the cells between *A* and *B* will have reached a maximum, and on account of their momenta the cells will continue turning, and produce a distortion of the chain in a reversed sense. At the same time the energy will, to a large extent, stream back from the region *mm* to the region *ee*, the ball *A* will be negatively charged, and the ball *B* will be positively charged. This reversed distortion of the chain of cells is then relieved by a reversed slip (a reversed current in the rods and gap), and so on.

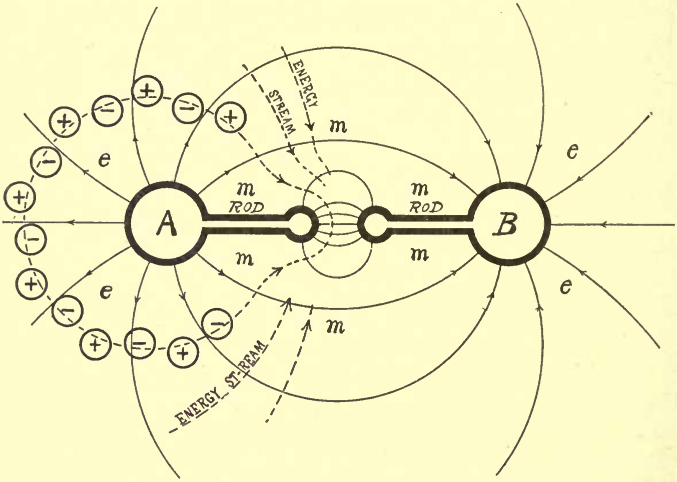

FIG. 8.

These oscillatory changes take place so rapidly that the portions of the distorted ether which are remote from *AB* do not follow the changes promptly. This gives rise to electrical waves, the nature of which at a distance from the vibrator is explained in Article 603.

CHAPTERS VII. AND VIII.

(*Revised.*)*

ELECTRIC CHARGE. ELECTROSTATICS.

19. Electric Charge.—The electric current in a wire may be looked upon as a transference of *electricity* or of *electric charge* along the wire, and unit current may be said to transfer unit charge per second so that

$$q = it \tag{1}$$

in which q is the amount of charge transferred in t seconds by a constant current i. If the current is not constant then one must consider the charge $\triangle q = i.\triangle t$ which is transferred in a short interval of time $\triangle t$, so that $\frac{\triangle q}{\triangle t} = i$ or

$$\frac{dq}{dt} = i. \tag{2}$$

That is, the current in a circuit is the rate, in units charge per second, at which charge is transferred along a wire.

Units of electric charge. The quantity of electric charge transferred in one second by a current of one ampère is called the *Coulomb;* q in equation (1) is expressed in coulombs when i is expressed in ampères and t in seconds. When, in equation (1) i is expressed in c. g. s. units of current and t in seconds, q is expressed in c. g. s. units of charge. One c. g. s. unit of charge is equal to ten coulombs.

Another independent unit of charge is defined later.

20. Measurement of electric charge.—When the electric charge to be measured is very large it may be determined by observing the time during which it will maintain a measured current. For example, a given battery, storage or primary, furnishes a definite number of coulombs of electricity before the materials of the battery are exhausted. Thus a given storage battery gives 10 ampères for 10 hours (100 ampère-hours, or 360,000 coulombs).

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* Articles, equations and figures are numbered in this revision independently of the numbers in the text-book.

The ballistic galvanometer. A small electric charge cannot be determined by measurement of current and time, as described above; such charges are measured by means of the ballistic galvanometer. The charge to be measured is sent as a quick rush of current through a galvanometer and the throw, or deflection of the needle is observed. This deflection d is proportional to the charge q which passes through the galvanometer, provided the whole of the charge passes through in a very short time, therefore

$$q = kd \tag{3}$$

in which k, the proportionality factor, is called the *reduction factor* of the galvanometer. This factor may be determined by observing the deflection produced by a known charge.

Theory of the Ballistic Galvanometer. See Text-book, Article 386.

21. Work done by an e. m. f. during the transferrence of a given charge.—Consider an e. m. f., E, pushing a current i through a circuit. The *rate* at which this e. m. f. does work is Ei which multiplied by a time t gives the work, W, done during that time, so that $W = Eit$; but $i.t$ is the charge transferred during the time t. Therefore

$$W = Eq \tag{4}$$

in which W is the work done by an e. m. f., E, in pushing charge q through a circuit.

22. Electrodynamics and Electrostatics.—The electric current is electricity in motion and the study of the phenomena of the electric current is called *electrodynamics*. This chapter and, to a certain extent, the succeeding chapters are devoted to the study of the phenomena of stationary electric charges. This study is called *electrostatics*.

23. Electrically charged bodies.—Consider two bodies of metal A and B, Fig. 9, connected as shown to the terminals of a battery, or to any source of e. m. f. When the wire is connected a momentary rush of electric current flows through the wire and the bodies *A* and *B* become *charged with electricity.* The body towards which the rush of current flows is said to become *positively charged* and the other *negatively charged.* Electrically charged bodies always occur thus in pairs, the positive charge on one body being always associated with negative charge on some other body or portion of a body.

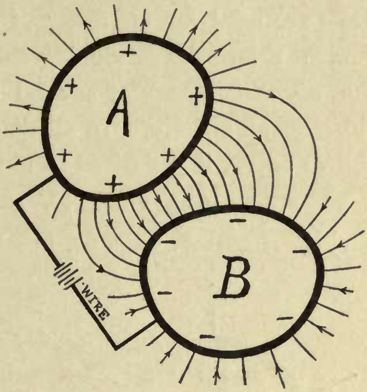

FIG. 9.

Electric field.—The region between the charged bodies *A* and *B*, Fig. 9, is theseat of a peculiar stress called *electric field,* similar in many respects to magnetic field; and the lines of force of this electric field trend somewhat as shown in the figure, touching the surface of both *A* and *B* at right angles. These lines of force are thought of as *going out from* the positively charged body and *coming in towards* the negatively charged body as indicated by the arrow heads.

Electrostatic attraction.—The charged bodies *A* and *B* attract; this attraction is called *electrostatic attraction.* This attraction shows that the lines of force of an electric field are in a state of tension and have a tendency to shorten. This tension of the lines of force of an electric field pulls outwards on the surface of *A* and of *B* at each point. This outward pull on the surface of a charged body is shown very strikingly by pouring a liquid over the sharp lip of a charged ladle. The liquid is pulled into very fine jets by the lines of force which emanate from the lip. When melted rosin is used in this way these jets congeal into very fine fibers which float about in the air.

Need of very large electromotive forces.—Electrostatic phenomena are easily perceptible only when bodies are charged with very large electromotive forces. Thus the phenomena described above are noticeable only when e. m. f.'s of many thousands of volts are used for charging *A* and *B*, Fig. 9. The most convenient means for producing these large e. m. f.'s is by means of the *electrical machine*, several forms of which are described in subsequent articles. In all diagrams, however, a battery will be shown where it is desired to indicate an e. m. f.

24. Mechanical analogue of charged bodies.—Consider two cavities *A* and *B*, Fig. 10, in an elastic solid, jelly, for example. If the cavity *A* is expanded, for example, by forcing air into it, and the cavity *B* contracted, the surrounding jelly will be stressed, the lines of stress being somewhat as indicated in the figure. It is not worth while to examine minutely into the stressed condition of the jelly in this case inasmuch as this stress is not completely analogous to electrical field. This matter is taken up again in Chapter XVI.

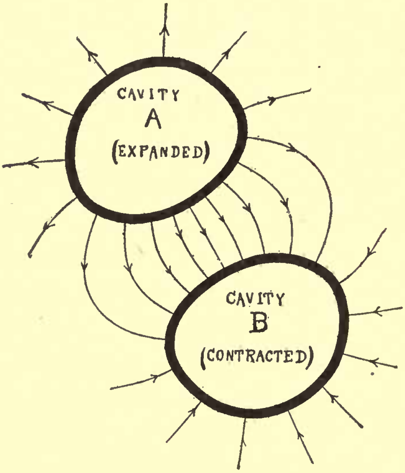

FIG. 10.

25. The electric discharge.—When charged bodies loose their charge they are said to be *discharged*. Many important and interesting phenomena accompany this discharging of bodies. These phenomena are described in Chapter X. For the purpose of the present discussion it is sufficient to note, that charged bodies when left to themselves always loose their charge more or less rapidly so that in electrostatic experiments charged conductors must be thoroughly insulated; and that two charged bodies, for example *A* and *B*, Fig. 9, loose their charges almost instantaneously when they are connected by a wire, and of course the electric field disappears at the same time. A line of force never begins and ends on the same conductor nor upon two conductors which are brought into contact or connected by a wire.

26. Electric charge resides wholly on the surface of a charged conductor.—*Electrical screening.* The electrostatic phenomena exhibited by the charged conductors *A* and *B*, Fig. 9, are precisely the same whether the bodies are solid or hollow; and, if the bodies are hollow, not the slightest effect of the charges can be detected inside of them, however thin their walls may be. *The lines of force of the electric field end, therefore, at the surface of the charged conductors* or, as it may be said, *the electric charges reside wholly on the surfaces of charged conductors.*

A conducting shell (for example a metal box) screens its interior completely so that no action of any kind reaches the interior from charged bodies outside. Thus a shell of metal *C*, Fig. 11, screens its interior completely. The lines of force which touch the shell *C* end at its surface. The *ending* on *C* of the lines of force from *A* is *negative charge*, and the *beginning* on *C* of lines of force which reach *B* is *positive charge.*

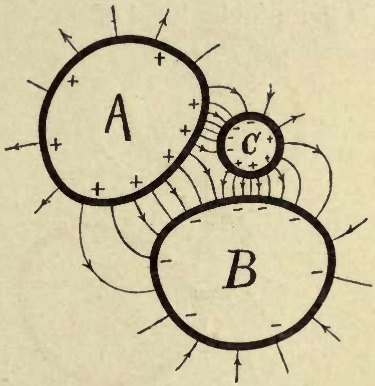

FIG. 11.

The fact that electrical field does not penetrate into a conductor shows that conductors cannot sustain the peculiar kind of stress which constitutes electrical field and therefore this electrical stress cannot be transmitted from the outside to the inside of a metal box.

27. Mechanical analogue of electrical screening.—Consider a solid body *B*, Fig. 12, entirely separated from the surrounding solid by the empty space *eee*. Stress and distortion of the surrounding solid cannot affect *B* in any way and, conversely, stress and distortion of *B* cannot affect the surrounding solid, for the empty space is incapable of transmitting the stress. This empty space in its behavior towards mechanical stresses is analogous to a conductor in its behavior towards electrical stresses (electric field).

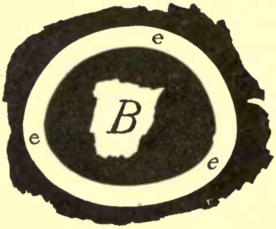

FIG. 12.

28. A charged conductor shares its charge with another conductor placed in contact with it.—Fig. 13 shows the lines of force in the neighborhood of a charged conductor *A*. When another conductor *B* is placed in contact with *A* the lines of force arrange themselves as shown in Fig. 14. The charge which was initially on *A* spreads over both *A* and *B* as shown by the *ending* of the lines of force.

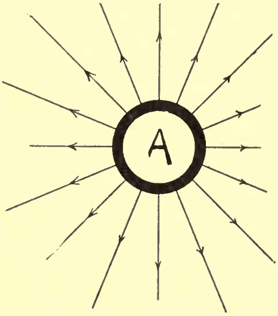

FIG. 13.

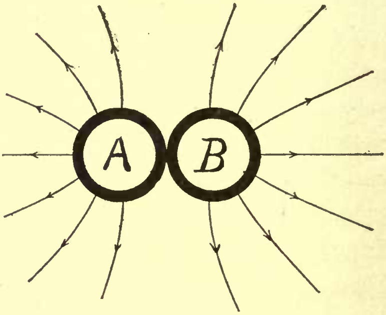

FIG. 14.

29. Faraday's experiment.—A charged body *B* is lowered into the interior of a metal vessel and the opening of the vessel is closed with a metal lid. As the body is lowered into the vessel, each line of force emanating from *B* is cut in two, as it were, by the wall of the vessel so that when *B* is entirely enclosed by the vessel *as many lines of force emanate from the external surface of the vessel as from the body B and all the lines of force which ema-*

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nate from B end on the internal surface of the vessel. Therefore if Q is the amount of charge on B, the amount of charge on the inner surface of the vessel is $-Q$ and the amount of charge on the outer surface of the vessel is Q. Fig. 15 shows the state of the electrical field while the body B is being lowered into the vessel and Fig. 16 shows the state of the electrical field when the body B is enclosed by the vessel and its lid.

Further, the distribution of the electric field outside of the vessel does not depend in any way upon the position of the body B inside of the vessel. Thus, in Figs. 16 and 17 the distribution of the electric field (lines of force) is very different inside while the external field is the same. If the body B (a conductor) is brought into contact with the wall of the vessel, the charges on the body and on the inner surface of the vessel disappear, while the external field is not affected, as indicated in Fig. 18.

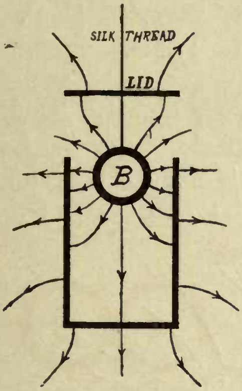

FIG. 15.

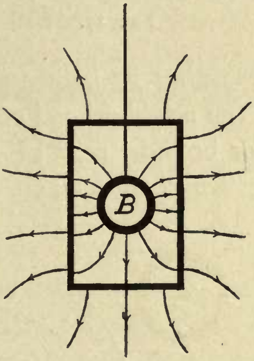

FIG. 16.

30. Giving up of entire charge by one body to another.—When the body B, Figs. 15, 16, 17 and 18, is lowered into the vessel and allowed to touch the interior it looses all its charge and remains without charge when removed from the vessel, while the charge left on the outside of the vessel is equal to and of the same sign as the original charge on B. The body B may thus be said to give up its entire charge to the vessel.

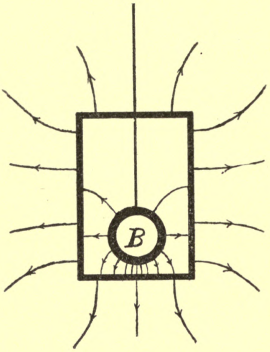

FIG. 17.

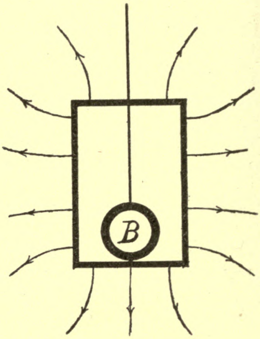

FIG. 18.

31. Charging by influence.—Let *A*, Fig. 19, be a charged body. The neighborhood of *A* is an electric field. Let *B* be a conductor brought near to *A*. This body *B* takes on positive and negative charges where the lines of force end upon it as shown. If a third conducting body *C* is brought into contact with *B* as shown in Fig. 20, then *B* and *C* are charged as indicated in the figure, and the bodies *B* and *C* retain these charges when they are separated and removed to a distance from *A*. This operation is called *charging by influence;* equal amounts of

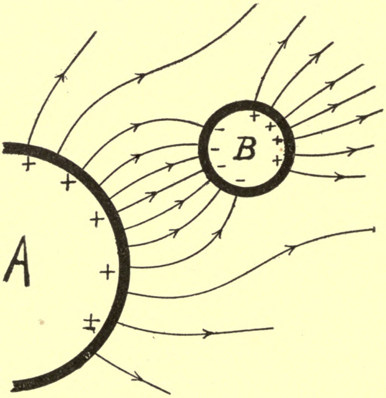

FIG. 19.

positive and negative charge being produced. It often occurs that one wishes only to charge the body *B*, in which case the hand may serve instead of the body *C*.

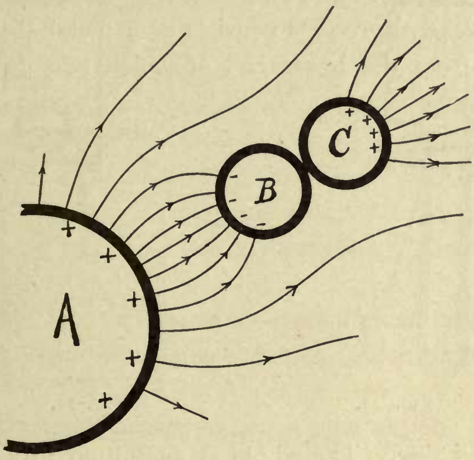

FIG. 20.

32. Charging by contact and separation.—Many substances when separated after intimate contact are found to be highly charged. Thus silk and glass when rubbed together and separated are charged, the glass positively and the silk negatively; fur and rosin when rubbed together and separated are charged, the fur positively and the rosin negatively. This charging by contact and separation is made use of in the so-called *Frictional electric machine*. Charging by contact and separation is more minutely discussed in a subsequent article.

33. Electroscopes.—A device for indicating the existence of an electric charge or of an electric field is called an *electroscope*.

The pith-ball electroscope. Electric charge on a body *AA*, Fig. 21, may be indicated by hanging from the body a light ball of pith as shown. The pith ball takes part of the charge of the body and the lines of force emanating from the ball pull it into the position shown. A suspended pith ball which has been (positively) charged is pulled in the direction of the lines of force when placed anywhere in an electric field ; this affords an easy means for tracing the lines of force. The pith ball used as an electroscope is usually gilded, making it a conductor, so that it may be easily charged and discharged.

The gold-leaf electroscope consists of a metal disk *D* and rod *R*, Fig. 22, from the lower end of which two gold leaves are hung side by side. The whole is supported in a glass case *cc* which protects the gold leaves from air currents. The sides of *cc* are lined with strips of metal foil *ff* to increase the sensitiveness. When the disk, rod and gold leaves are charged the leaves are pulled apart by the lines of force as shown in Fig. 23, which for clearness, shows the instrument without a glass case.

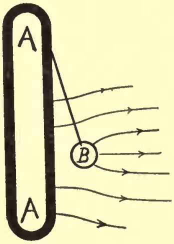

FIG. 21.

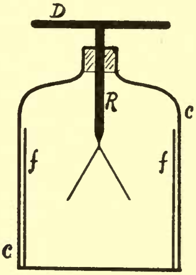

FIG. 22.

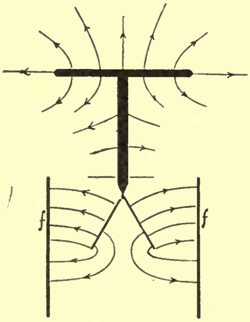

FIG. 23.

The action on the gold-leaf electroscope, of a charged body brought near to the plate *D* is briefly as follows:

(1) *When the electroscope has no initial charge* some of the lines of force pass from the body *into* the disk and then *out from* the leaves to the strips *ff*, causing the leaves to diverge. If the body is removed the electroscope again becomes neutral. If, while the charged body is near *D*, the disk or rod is touched with the hand the lines of force passing *out from* the leaves cease to exist and the leaves fall together. If now the charged

body is removed the lines of force going into the disk from the charged body spread over disk, rod and leaves, the leaves diverge, and the electroscope is left charged.

(2) *When the electroscope has an initial charge*, say a positive charge, then a positively charged body brought near to *D* pushes the initial charge down into the leaves, as it were, and the divergence of the leaves is increased. If a negatively charged body is brought near to *D* the positive charge is pulled up into the disk, as it were, and the divergence of the leaves is decreased. If the negatively charged body is brought nearer, the leaves will come together; and if the body is brought still nearer, the leaves will again diverge.

This action of a charged body upon a gold-leaf electroscope affords a convenient means for detecting and identifying positive and negative charges.

34. The electrophorus and electrical machines.—See text-book, Arts. 438, 439, 440, 441, 442, and 443.

35 The condenser. Electrostatic capacity.—Consider two parallel metal plates *AB*, Fig. 24, separated by air and connected through a ballistic galvanometer to the terminals of a battery as shown. When the connection with the battery is made electric charge rushes through the wire into the plates as described in article 23. The quantity of charge which thus rushes into the plates, and which may be measured by the ballistic galvanometer *G*, Fig. 24, is strictly proportional * to the e. m. f. of the battery. That is

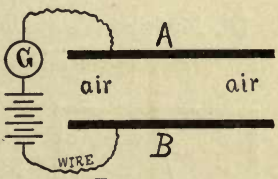

FIG. 24.

$$Q = JE \tag{5}$$

in which Q is the quantity † of electric charge which is pushed into a pair of insulated plates by an e. m. f., E, and J is the proportionality factor. This arrangement of metal plates is called a *condenser*, and the factor J is called the *electrostatic capacity* or simply the *capacity* of the condenser. When the dielectric between the plates of a condenser is air, the condenser is called an *air condenser.*

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* An experimental fact.

† The quantity which is on one plate.

Remarks: The charge which is pushed into any pair of bodies, as for example AB, Fig. 9, is proportional to the e. m. f. so that equation (5) is entirely general, and any pair of bodies constitutes a *condenser.*

Units of capacity. A condenser is said to have one *farad* of capacity when one coulomb of charge can be pushed into the condenser by an e. m. f. of one volt; J in equation (5) is expressed in farads when Q is expressed in coulombs and E in volts. The farad is very large compared with the capacities of condensers met with in practice, therefore the *micro-farad* (one millionth of a farad) is a more convenient unit.

When, in equation (5), Q is expressed in c. g. s. units charge and E in c. g. s. units e. m. f., J is expressed in c. g. s. units capacity. The c. g. s. unit of capacity is equal to 10^9 farads.

The Leyden Jar is a condenser made by coating the inside and outside of a glass jar with metal foil.

36. Hydrostatic analogue of the condenser.—Consider a chamber with water-tight compartments, A and B, Fig. 25, separated by an elastic diaphragm DD of rubber. If a pump P is connected to the compartments as shown a definite quantity of water Q will be forced through the pipe out of one compartment into the other and this quantity will be proportional to the difference of pressure E generated in A and B by the pump. That is

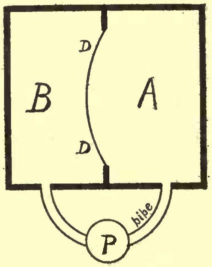

FIG. 25.

$$Q = JE \qquad (5) \text{ bis}$$

in which J is the proportionally factor. The diaphragm separating A and B is subjected to mechanical stress very much as the insulator or dielectric between the plates of a condenser is subject to electrical stress.

37. Measurement of capacity.—The capacity of a condenser may be determined by measuring with the ballistic galvanometer the charge which is pushed into the condenser by a known e. m. f. The capacity may then be calculated from equation (5).

In article 47 an equation is derived expressing (in new units) the capacity of an air condenser in terms of distance and size of the plates. Capacities of air condensers may be calculated from this equation.

38. Inductivity of dielectrics.—The insulating substance between the plates of a condenser, or between any electrically charged bodies is called the *dielectric*. The capacity of a condenser (given size of plates at given distance apart) depends upon the dielectric. The quotient: *capacity of a condenser with given dielectric divided by its capacity with air between its plates* is called the *inductivity** of the dielectric. For example, the inductivity of kerosene is 2.04; that is the capacity of a condenser is 2.04 times as great with kerosene between its plates as it is with air between its plates. A table, page 141 text-book, gives the inductivities of a few substances.

39. Work done in charging a condenser.—When an e. m. f. E is connected to an uncharged condenser, the work EQ done by the e. m. f. as the condenser is charged is in part dissipated because of the rush of charge into the condenser which is followed by a surging of the charge back and forth. In order that the whole work done by an e. m. f. in charging a condenser may be used in forcing the charge into the condenser without any dissipation, the e. m. f. must have a value zero when it is connected to the condenser and be made to increase slowly in value. The work done in charging a condenser in this way is all represented in the potential energy of the charged condenser. The potential energy W of a charged condenser is

---

* Sometimes called *specific inductive capacity.*

$$W = \tfrac{1}{2}\frac{Q^2}{J} \tag{6}$$

or

$$W = \tfrac{1}{2}EQ \tag{7}$$

or

$$W = \tfrac{1}{2}JE^2. \tag{8}$$

Proof: Let $q\ (= Je)$ be the charge in the condenser when the growing e. m. f. has reached the value e, and let $\triangle q$ be the additional charge which is pushed into the condenser by a slight increase of the charging e. m. f.; then $\triangle W = e \cdot \triangle q$ is the work done in producing this increase of charge; but $e = \frac{q}{J}$ from equation (5) so that

$$\triangle W = \frac{1}{J} q \cdot \triangle q.$$

Integrating this expression from $q = 0$ to $q = Q$ we have

$$W = \tfrac{1}{2}\frac{Q^2}{J}.$$

The equations (7) and (8) are obtained by writing JE for Q and J^2E^2 for Q^2, respectively, in equation (6).

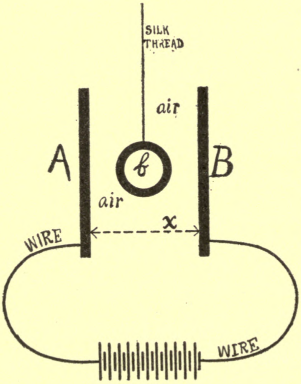

FIG. 26.

40. Electric field and its intensity. *Transference of charge by a moving ball.*—Consider a condenser AB, Fig. 26, with a metal ball suspended between its plates as shown. When the condenser is charged with high e. m. f. this ball vibrates back and forth, striking one plate and then the other, and each movement of the ball transfers a definite charge q from plate to plate. The work done on the ball by the force F pushing it across from plate to plate through the distance x is Fx and this work is furnished by the battery as it replenishes the charge. The work done by the battery is Eq. Therefore, $Fx = Eq$ or

$$F = \frac{E}{x} \cdot q. \tag{9}$$

Any region in which a charged body is acted upon by a force * is called an *electric field* and the intensity of an electric field at a point is the factor which, multiplied by the charge at that point, gives the force acting on the charged body. Comparing this statement with equation (9) we see that the intensity f of the electric field between the plates of the condenser is

$$f = \frac{E}{x} \tag{10}$$

in which E is the e. m. f. between the condenser plates, and x is the distance of the plates apart. Writing f for $\frac{E}{x}$ in equation (9) we have the general equation

$$F = fq \tag{11}$$

in which F is the force acting upon charge q, when the charge is placed at a point in an electric field, and f is the intensity of the field at the point.

The *direction* of an electric field at a point is taken as the direction of the force which would act on a positive charge placed at the point.

See Arts. 341 to 344 of the text-book for statements concerning magnetic field which hold for electric field.

41. General Relations between electromotive force and Electric Field.—Consider two conductors A and B, Fig. 27, charged by a battery of e. m. f. E connected as shown. Suppose that an amount q of charge is transferred from A to B along the path pp'. The work done by the battery in replenishing the charge is

$$W = Eq, \tag*{(4) bis}$$

and *this is equal to the work done by the forces with which the electric field acts upon the charge while the charge is passing along*

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* That is a force which depends upon the charge, and does not exist when the body has no charge.

the path. Consider an element $\triangle s$ of the path. Let f be the field intensity at this element and ε the angle between $\triangle s$ and f, see figure. Then qf is the total force acting on the charge q as it is passing along the element $\triangle s$, $qf \cos \varepsilon$ is the resolved part of this force in the direction of the element, and $qf \cos \varepsilon \,.\, \triangle s$ is the work done by this force as the charge moves over the element. Therefore, the total work done by the electric field upon the charge q as it is carried from p to p' is $W = \Sigma qf \cos \varepsilon \,.\, \triangle s$ or

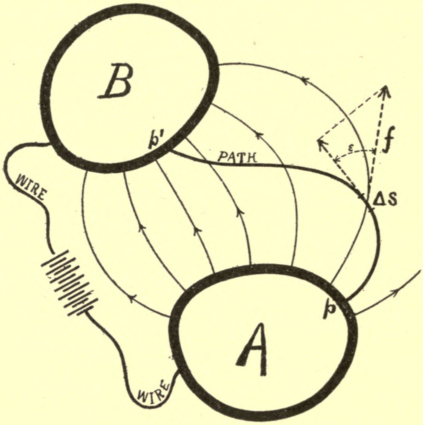

FIG. 27.

$$W = q\Sigma f \cos \varepsilon .\ \triangle s \tag{12}$$

Comparing this with the equation $W = Eq$ (4), we see that the e. m. f. along a given path in an electric field is

$$E = \Sigma f \cos \varepsilon .\ \triangle s. \tag{13}$$

That is, each element of the path being multiplied by the resolved part parallel to the element of the electric field intensity at the element, the sum of these products is the e. m. f. along the path. The equation $f = \frac{E}{x}$ (10) or $E = fx$ as applied to the electric field between parallel plates is a special case of equation (13).

42. Electric Potential.—The e. m. f.* between two points in an electric field is called the *difference of potential** between those points. If a point or a region (for example the earth) be chosen arbitrarily as the region of *zero potential* then the *electric potential* at a point* may be defined as the e. m. f. between that point and the arbitrarily chosen point or region of zero potential. Electric potential is in some respects analogous to hydrostatic pressure and is often spoken of as *electric pressure.*

Surfaces of equipotential. The e. m. f. is zero along a path which is everywhere perpendicular to the lines of force of an electric field. Therefore the potential has the same value at all points of such a path. Similarly the potential has the same value at all points of a surface *which is everywhere at right angles to the lines of force;* such a surface is called an *equipotential surface.* The

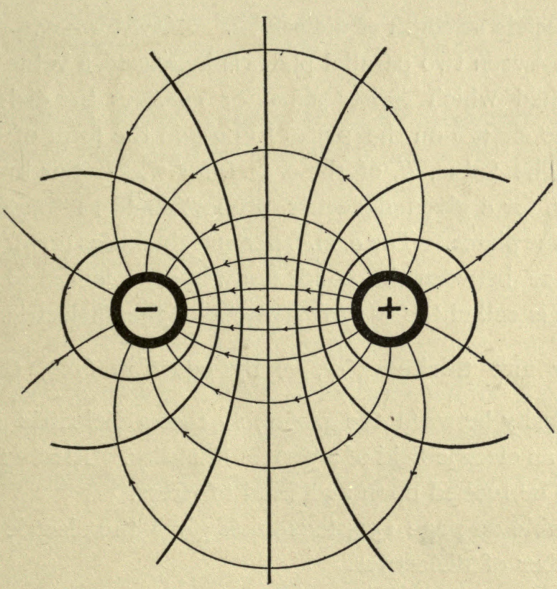

FIG. 28.

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*When the e. m. f. between two points is not the same for different paths connecting the points then one cannot speak of e. m. f. as a difference of potential. This matter is fully discussed in Chapter I. of the text-book.

lines of force in an electric field touch the surfaces of conductors at right angles, as has been pointed out in article 23, so that the potential has the same value over the entire surface of (and throughout) a charged conductor.

The heavy lines in Fig. 28 show the approximate trend of the surfaces of equipotential in the region surrounding two charged spheres, the lines of force are marked by the arrow heads. In the neighborhood of an isolated charged sphere the surfaces of equipotential are spherical surfaces concentric with the charged sphere.

43. Electric strength of dielectrics. *Electric stress.*—When the e. m. f. between two parallel plates is increased, a value is eventually reached which *breaks down* or *ruptures* the dielectric and allows the charge on the plates to pass in the form of an *electric spark.* The value, E, of the e. m. f. for which this break down occurs is, for a given dielectric, proportional* to the distance x between the plates. The e. m. f. per unit thickness (e. m. f., divided by distance between the condenser plates) required to rupture a dielectric is called the *electric strength* of the dielectric. This e. m. f. per unit thickness, $\frac{E}{x}$, is, by equation (10), the electric field intensity between the plates, so that a dielectric is broken down by an electric field of a certain intensity. Electric field may therefore be looked upon as a kind of stress.

The table on page 152, text-book gives the electric strengths of a number of dielectrics.

44. Electric Flux.—The product of the intensity of an electric field into an area at right angles to the direction of the field is called the electric flux across the area. That is

$$N = fa \tag{14}$$

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* When any two bodies, as A and B, Fig. 9, are charged by an increasing e. m. f., the dielectric breaks down when the e. m. f. reaches a certain value, but the e. m. f. required to produce rupture is proportional to the thickness of the dielectric only in case of parallel plates.

in which N is the electric flux across area a at right angles to a field of intensity f. The unit electric flux is the flux across unit area at right angles to unit field. This unit flux is called the *line*. See articles 5 and 6 of this pamphlet for statements concerning magnetic flux which hold for electric flux as well.

The electric flux across an extended surface in an electric field cannot be expressed as the product of the field intensity into the area of the surface as in equation (14), for the reason that the field will in general vary in intensity from point to point and also for the reason that the field may not be at right angles to the surface at every point. In this case we have

$$N = \Sigma f. \cos \varepsilon. \Delta S \tag{15}$$

in which ΔS is an element of the surface, f is the field intensity at the element, $f \cos \varepsilon$ is the resolved part of f perpendicular to ΔS, and $f \cos \varepsilon. \Delta S$ is the flux across the element.

45. Dependence of the capacity of an air condenser upon size and distance of plates.—The capacity, J, of an air condenser is proportional to $\frac{a}{x}$, where a is the area of one plate and x is the distance between the plates. That is

$$J = C\frac{a}{x} \tag{16}$$

in which C is a constant. When J is expressed in c. g. s. units capacity, a in square centimeters and x in centimeters the experimentally determined value C is $\frac{1}{4\pi v^2}$ where $v = 3 \cdot 10^{10} \frac{\text{cm.}}{\text{sec.}}$.*

If the capacity of an air condenser is to be expressed in farads we have

$$J_{\text{farads}} = 885 \cdot 10^{-16} \frac{a \text{ cm.}^2}{x \text{ cm.}}. \tag{17}$$

If the condenser has between its plates a dielectric of which the inductivity is k, see article 38, then equation (16) becomes

$$J = kC\frac{a}{x}. \tag{18}$$

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* This is precisely the velocity of light. The significance of this fact is explained in Chapter XV.

46. Amount of electric flux from an electric charge.—Consider a parallel plate air condenser of capacity J. Substituting the value of $J\left(=\frac{Q}{E}\right)$ from equation (5) in equation (16) we have

$$Q = Ca\frac{E}{x}\cdot \tag{19}$$

Now, $\frac{E}{x}$ is the electric field intensity between the plates (direction of field perpendicular to the plates) and $a\frac{E}{x}$ is the electric flux N from plate to plate, according to equation (14). Therefore

$$Q = CN \tag{20}$$

in which C is the constant mentioned in article 45. Therefore the *electric flux outwards from charge Q is proportional to the charge, and the charge on a body may be expressed in terms of the number of lines of electric flux which terminate on the body.*

47. A new set of electrical units.—The factor C in equations (16), (19) and (20) occurs in many equations relating to electric charge and electric field. The discussion of electrostatics is somewhat simplified by so choosing the various electrical units as to give to C the value $\frac{1}{4\pi}\cdot$ In this case equation (20) becomes

$$N = 4\pi Q \tag{21}$$

equation (16) becomes

$$J = \frac{a}{4\pi x} \tag{22}$$

and equation (18) becomes

$$J = \frac{ka}{4\pi x}\cdot \tag{23}$$

That is, *the electric flux passing out from charge Q is equal to $4\pi Q$, and the capacity of an air condenser is equal to $\frac{a}{4\pi x}$ in which* *a is the area of the plates* (*one plate*) *and x is the distance between them,* provided the various electrical quantities are expressed in terms of the **new units.**

These new units are to be chosen so that the factor C may have the value $\frac{1}{4\pi}$ instead of the value $\frac{1}{4\pi v^2}$; that is, *this factor is to be made v^2 times as large.* By inspection of equation (19) this is seen to require any one of the following changes in the units of charge and e. m. f.:

(*a*) The number which expresses a given e. m. f. must be made v^2 times as small, that is, the unit e. m. f. must be chosen v^2 times as large as the unit heretofore used, unit charge being unchanged; or

(*b*) *The number which expresses a given e. m. f. must be made v times as small and the number which expresses a given charge must be made v times as large. That is the unit e. m. f. must be chosen v times as large and the unit charge v times as small as the units heretofore used;* or

(*c*) The number which expresses a given charge must be made v^2 times as large, that is the unit charge must be made v^2 times as small as the unit heretofore used, unit e. m. f. being unchanged.

So far as the immediate object, of reducing the value of the factor C to $\frac{1}{4\pi}$, is concerned (*a*), (*b*) and (*c*) are equally satisfactory; but another consideration is important, namely, that the work, Eq [see, equation (4)], done by an e. m. f. in transferring a charge may still be equal to the product Eq where e. m. f. and charge are expressed in terms of the new units. The scheme (*b*) satisfies this condition inasmuch as the numerical value of a charge is increased in the same ratio that the numerical value of an e. m. f. is decreased. That is the numerical value of the product Eq is not affected by this change of units. A simple and direct definition of the new unit charge will be given later. For brevity these new electrical units (which are also c g. s. units) will be called *Faraday units* to distinguish them from the units which have been heretofore, and are to be hereafter, spoken of as c. g. s. units.

Concerning the Faraday unit of capacity, it is seen to be of the same dimensions as *length* from equation (22); a given capacity is therefore expressed as a *length*, and the unit (Faraday's unit) capacity is called the *centimeter*.

Remarks: Many equations in magnetism require the introduction of the factor C [equation (16)] when the quantities such as pole strength, magnetic field intensity, etc., are expressed in Faraday units.

Relative values of Faraday units and units heretofore used:

One Faraday unit charge $= \frac{1}{v}$ c. g. s. units charge

$= \frac{10}{v}$ coulombs,

One Faraday unit current $= \frac{1}{v}$ c. g. s. units current

$= \frac{10}{v}$ ampères,

One Faraday unit magnetic field $= \frac{1}{v}$ c. g. s. units magnetic field,

One Faraday unit e. m. f. $= v$ c. g. s. units e. m f.

$= \frac{v}{10^8}$ volts,

One Faraday unit electric field $= v$ c. g. s. units electric field

$= \frac{v}{10^8}$ volts per centimeter,

One Faraday unit magnetic pole $= v$ c. g. s. units magnetic pole,

One Faraday unit capacity $= \frac{1}{v^2}$ c. g. s. units capacity

$= \frac{10^9}{v^2}$ farads,

One Faraday unit resistance $= v^2$ c. g. s. units resistance

$= \frac{v^2}{10^9}$ ohms,

$$\text{where } v = 3 \times 10^{10}\frac{\text{cm.}}{\text{sec.}}.$$

If the student wishes to look more minutely into this matter of electrical units he will find a full discussion of it in Chapter XV.

48. The spherical condenser.—Consider two concentric metal spheres A and B, Fig. 29. Let R be the external radius of A and R' the internal radius of B. If the radius of B is very large the sphere A is said to stand by itself or to be isolated. Let the spheres be charged (charge on A being $+Q$ and on inside of B, $-Q$) by a battery of e. m. f. E connected to A and B.

It is desired to find the intensity, at each point, of the electric field in the region between A and B, and to find the relation between R, R', E and Q.

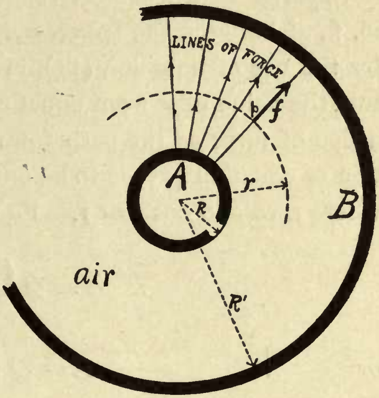

FIG. 29.

Electric field intensity near a charged sphere. The lines of force of the electric field between A and B are straight lines radiating from A to B as shown in Fig. 29. Let f be the electric field intensity at the point p distant r from the center of A. Describe a spherical surface of radius r, concentric with A. The electric flux across this spherical surface is $4\pi r^2 . f$ according to equation (14), $4\pi r^2$ being the area of the spherical surface. But this total flux out from the charge Q which is on A is equal to $4\pi Q$ from equation (21) so that $4\pi Q = 4\pi r^2 f$ or

$$f = \frac{Q}{r^2}. \tag{24}$$

This equation expresses the electric field intensity at a point distant r from the center of the sphere A. The value of the radius of the sphere A does not affect the validity of equation (24) so that this radius may be supposed to be very small, in which case the charge Q is called a *concentrated charge*. Equation (24) therefore expresses the electric field intensity at a point distant r from a concentrated charge Q. The conception of concentrated charge is of the same importance in the study of electrostatics as is the conception of concentrated magnetic pole in the study of magnetism.

Capacity of a spherical condenser. An expression for the e. m. f., E, between the spheres A and B, Fig. 29, can be obtained by the use of equation (13), in terms of Q, R and R'. Substitute the value of f from equation (24) in equation (13). Choose a line of force as the path from A to B over which the summation of equation (13) is to be extended, then the angle ε is everywhere zero and $\cos \varepsilon = 1$. Equation (13) then becomes:

$$E = Q\int_R^{R'} \frac{1}{r^2} \cdot dr$$

or

$$E = Q\left(\frac{1}{R} - \frac{1}{R'}\right)$$

or

$$Q = \frac{1}{\frac{1}{R} - \frac{1}{R'}} . E \tag{25}$$

Comparing this with equation (5) we find that the capacity, J, of the concentric spheres is

$$J = \frac{1}{\frac{1}{R} - \frac{1}{R'}} . \tag{26}$$

If the external sphere is indefinitely large so that the sphere A stands alone or isolated then $\frac{1}{R'}$ is negligibly small and equation (26) becomes

$$J = R. \tag{27}$$

That is, the capacity of an isolated sphere in air is equal to its radius.

49. Electrostatic attraction of concentrated charges.—Consider a charge Q_2 at a distance r from a charge Q_1, Fig. 30. The electric field intensity at Q_2 due to Q_1 is $f = \frac{Q_1}{r^2}$ (24) and the charge Q_2 is acted upon by a force which is equal to $Q_2 f$ according to equation (11), so that the force with which the two charges repel (or attract) is

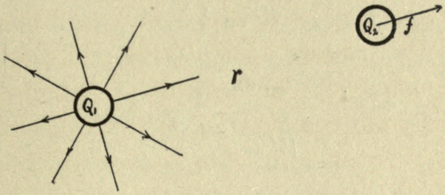

FIG. 30.

$$F = \frac{Q_1 Q_2}{r^2}. \tag{28}$$

The fact expressed by this equation, namely that two charges attract or repel each other with a force which is inversely proportional to the square of the distance between the charges, was discovered by Coulomb and is called *Coulomb's Law.* When Q_1 and Q_2 are both positive or both negative their product is positive and they repel each other. When one is positive and the other negative their product is negative and they attract. Therefore when F, equation (28), is positive it is a repulsion and when it is negative it is an attraction.

The Faraday unit charge. The relation expressed by equation (26) furnishes the simplest basis for the definition of the unit charge (Faraday unit). *The unit charge is that charge which repels an equal charge at a distance of one centimeter with a force of one dyne.*

50. Electrostatic attraction of parallel plates.—Consider two parallel plates of area a, at distance x apart, with air between, and charged by an e. m. f., E, so that

$$Q = \frac{a}{4\pi x} E \tag{29}$$

in which $\frac{a}{4\pi x}$ is the capacity of the condenser in Faraday units, equation (22). The energy of this charged condenser is

$$W = \tfrac{1}{2}\frac{Q^2}{J} \tag{7 bis}$$

which, since $J = \frac{a}{4\pi x}$, becomes

$$W = \frac{2\pi Q^2}{a} \cdot x. \tag{30}$$

If the distance x between the plates is increased by the amount $\triangle x$ while the plates are insulated so that Q cannot change, then the energy of the condenser increases by the amount

$$\triangle W = \frac{2\pi Q^2}{a} \cdot \triangle x$$

and this increase of energy comes from the work done in pulling the plates apart against their electrostatic attraction. Let F be the force of attraction of the plates, then $F.\triangle x$ is the work done in increasing their distance by $\triangle x$, so that

$$F.\triangle x = \triangle W = \frac{2\pi Q^2}{a} \cdot \triangle x$$

or

$$F = \frac{2\pi Q^2}{a} \tag{31}$$

in which F is the force of attraction of parallel air condenser plates of area a, when charged; the charge on one plate being $+Q$ and the charge in the other being $-Q$. It is remarkable that the force of attraction is independent of the distance between the plates for *given charge*, the plates being large compared to the distance between them.

By using the values $J = \frac{ka}{4\pi x}$ (23) in the above discussion we find

$$F = \frac{2\pi Q^2}{ka} \tag{32}$$

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in which F is the force of attraction of condenser plates of area a in a dielectric of which the inductivity is k. Thus condenser plates with *given charge* attract with $\frac{1}{2{\cdot}04}$ as much force in petroleum as in air.

An expression for the force of attraction of parallel plates in terms of the e. m. f. between them may be found by substituting for Q its value $\frac{a}{4\pi x} \cdot E \left(\text{or } \frac{ka}{4\pi x} \cdot E\right)$ in equation (31) (or equation (32)). This gives for an air condenser

$$F = \frac{a}{8\pi} \frac{E^2}{x^2} \tag{33}$$

and for a condenser with any other dielectric

$$F = \frac{ka}{8\pi} \cdot \frac{E^2}{x^2}. \tag{34}$$

Thus the electrostatic attraction of condenser plates with *given e. m. f.* is 2.04 times as great in petroleum as it is in air.

It can be shown that *the force of attraction of any two charged bodies is proportional to the square of the e. m. f. between them.* The case of parallel plates is one of the very few cases in which the value of the proportionating factor can be calculated in terms of the dimensions and distance of the bodies. The equation (33) is used in connection with the *Absolute Electrometer;* and the fact that any two charged bodies attract with a force which is proportional to the square of the e. m. f. between them is taken advantage of in the *Quadrant Electrometer.*

51. The absolute electrometer.—If the force of attraction of the plates of a parallel-plate air condenser is measured the e. m. f. between the plates may be calculated from equation (33). This equation gives the e. m. f. in Faraday units; one Faraday unit e. m. f. being equal to 300 volts. The *absolute electrometer* is an instrument for measuring e. m. f. in this way. The absolute electrometer is described in article 482, text-book.

52. The quadrant electrometer.—The absolute electrometer can be used for measuring large e. m. f.'s, only, inasmuch as the force of attraction of two charged plates is too small to measure by a balance when the e. m. f. is small. For example the attraction of two plates each of 133 cm.^{2} area, at a distance of one cm. is about one ten-thousandth of a dyne or about one ten-millionth of the weight of a gram for an e. m. f. of one volt. For the measurement of small e. m. f.'s by electrostatic attraction the *Quadrant Electrometer* is used. The quadrant electrometer is described in Art. 383, text-book. The two schemes for connecting the various parts of a quadrant electrometer are as follows:

First arrangement.—In this arrangement one terminal of the e. m. f., e, to be measured is connected to the needle and to the quadrants q_2q_2, Fig. 268, and the other terminal of the e. m. f. is connected with the quadrants q_1q_1. In this case the force tending to turn the needle is proportional to e^2, and the deflection d of the needle is sensibly proportional to e^2, or e is proportional to $\sqrt{d}$ so that

$$e = k\sqrt{d}. \tag{35}$$

With this arrangement e. m. f.'s as small as, say, 10 volts may be measured. This arrangement is always employed when the electrometer is used to measure the e. m. f. of an alternating dynamo.

Second arrangement.—In this arrangement one terminal of the e. m. f., e, to be measured is connected to the quadrants q_2q_2, Fig. 268, and the other terminal to the quadrants q_1q_1; and a very large auxiliary e. m. f., E, is connected, one terminal to the needle and the other terminal to the quadrants q_1q_1. Then the e. m. f. between the needle and quadrants q_1q_1 is E and the needle is pulled towards these quadrants with a force kE^2 (proportional to E^2). The e. m. f. between the needle and quadrants q_2q_2 is $E + e$, so that the needle is pulled towards these quadrants with a force $k(E + e)^2$. The total force acting to deflect the needle is $k(E + e)^2 - kE^2 = 2kEe + ke^2$, but e^2 is negligible in comparison with Ee so that the deflecting force, and, therefore, the deflection d is sensibly proportional to Ee or to e, since E is supposed constant, so that

$$e = kd. \tag{36}$$

With this second arrangement e. m. f.'s as small as, say, 0.01 volt may be measured. The reduction factor k is, of course, entirely different in equations (35) and (36).

53. Energy of the electric field.—The whole of the energy of an electric charge resides in the electric field. Thus the energy of a parallel-plate air condenser is $W = \frac{1}{2}JE^2$ by equation (8); but $J = \frac{a}{4\pi x}$ by equation (22) so that

$$W = \frac{a}{8\pi x} \cdot E^2 = \frac{ax}{8\pi} \cdot \frac{E^2}{x^2}.$$

But ax is the volume of the region between the plates and $\frac{E}{x}$ is the intensity of the electric field between the plates so that

$$\begin{array}{r}\text{Energy per unit volume} \\ \text{of an electric field}\end{array} = \frac{1}{8\pi} f^2. \tag{37}$$

Tension along the lines of force in an electric field. The total pull of the plates of an air condenser is $F = \frac{a}{8\pi} \cdot \frac{E^2}{x^2}$ from equation (33), so that the pull per unit area is $\frac{1}{8\pi} \frac{E^2}{x^2}$, but $\frac{E}{x}$ is the intensity of the electric field, so that the pull per unit area is $\frac{1}{8\pi} f^2$. This pull is necessarily transmitted through something between the plates, else the plates could not act on each other. Therefore

$$\begin{array}{r}\text{The pull per unit area} \\ \text{of an electric field}\end{array} = \frac{1}{8\pi} f^2. \tag{38}$$

The curvature of the lines of force of an electric field as exhibited in Fig. 9 shows, since the lines of force tend to shorten, that they also push sidewise on each other. It can be shown that this side push per unit area is equal to the pull in the direction of the lines of force, namely, $\frac{1}{8\pi}f^2$.

54. Contact e. m. f.—When any two substances are left in contact they settle to a state of equilibrium* *with a definite e. m. f. between them.* This e. m. f. is called the *contact e. m. f.* of the substances.

55. Charging by contact and separation.—Consider plates of zinc and copper at a distance x apart. When the plates have settled to equilibrium there will be an electric field between them of which the intensity is $\frac{E}{x}$ where E is the contact e. m. f. of zinc and copper. The lines of force of this field are shown in Fig. 31. If the plates are connected by a wire and moved apart, the e. m. f. between them will remain equal to E and the field intensity between them will, of course, decrease as x increases. If, however, the plates are insulated the electric field intensity between them does not decrease as they are moved apart inasmuch as the charges cannot escape and given charges signify a fixed number of lines of force ending on the plates. Therefore, the e. m. f. between the plates increases in proportion to the increase of x. This production of large e. m. f.'s by separating bodies from very near juxtaposition is called *charging by contact and separation.*

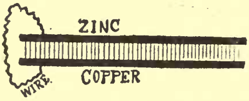

FIG. 31.

Example. The contact e. m. f. of zinc and copper is about 0.9 volt. When zinc and copper plates settle into equilibrium at a distance of, say, 0.01 cm. apart, the field intensity between them is 90 volts per cm. If now the plates are insulated and separated to a distance of 10 cm. the e. m. f. between them will be 900 volts.

The phenomenon of charging by contact and separation seems to be exhibited by all substances. Thus glass and silk when rubbed together and separated are charged, the glass positively and the silk negatively; rosin and fur when rubbed together and separated are charged, the rosin negatively and the fur positively.

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* That is to say a state in which there is no tendency to further change of any kind.

When metal plates are charged by contact and separation great care must be exercised to prevent the plates from touching after they are slightly separated, for this will allow the e. m. f. between the plates to drop immediately to the value of the contact e. m. f. No such difficulty exists in the charging of non-conductors by contact and separation.

INDUCTANCE.

56. The Magnetic Field as a Seat of Kinetic Energy.—The magnetic field is a kind of obscure motion of the all-pervading medium, the ether ; and this motion represents energy. The amount of energy* in a given portion of a magnetic field is proportional to the square of the intensity of the field. This is analogous to the fact that the kinetic energy of a portion of a moving liquid is proportional to the square of the velocity of the liquid.

57. Kinetic Energy of the Electric Current in a Coil. *Definition of Inductance.*—The kinetic energy of an electric current is the energy which resides in the magnetic field produced by the current. The field intensity is at each point proportional to the current and the energy at each point is proportional to the square of the field intensity, that is, to the square of the current. Therefore the total kinetic energy of the field is proportional to the square of the current. That is

$$W = \tfrac{1}{2}Li^2 \tag{39}$$

in which W is the total energy of a current i in a given coil, and

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* A mechanical conception of the magnetic field is given in Chapter XVI. The kinetic energy per unit volume (in air) of a magnetic field is $\frac{1}{8\pi}f^2$ where f is the intensity of the field. Compare article 53, this pamphlet, concerning energy (potential) per unit volume of an electric field.

($\frac{1}{2}L$) is the proportionality factor. The quantity L is called the *inductance* of the given coil.

Units of inductance. When, in equation (39), W is expressed in joules and i in ampères, then L is expressed in terms of a unit called the *henry*. When W is expressed in ergs and i in c. g. s. units of current, then L is expressed in c. g. s. units of inductance. This c. g. s. unit of inductance is called the *centimeter*, for the reason that the square of a current must be multiplied by a length to give energy or work; that is, inductance is expressed as a length and the unit of inductance is of course the unit of length. The henry is equal to 10^9 centimeters of inductance.

Example. A given coil with a current of 0.8 c. g. s. units produces a magnetic field of which the total energy is 6,400,000 ergs, so that the value of L for this coil is 20,000,000 centimeters. If current is expressed in ampères and energy in joules then the total energy corresponding to 8 ampères would be 0.64 joules and the value of L would be 0.02 henry.

58. Noninductive circuits.—A circuit of which the inductance is negligibly small is called a *noninductive circuit*. Since the inductance of a circuit depends upon the energy of magnetic field, therefore a noninductive circuit is one which produces only a weak magnetic field, or a magnetic field which is confined to a very small region. Thus the two wires, Fig. 32, constitute a noninductive circuit, especially if they are near together; for, these two wires with opposite currents produce only very feeble magnetic field in the surrounding region. The wires used in resistance boxes are usually arranged noninductively. This may be done by doubling the wire back on itself, as in Fig. 32, and winding this double wire on a spool. In this case the e. m. f. between adjacent wires at *ab*, Fig. 32, is great and the wires have considerable electrostatic capacity; or the wire may be wound in

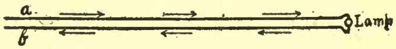

FIG. 32

one layer on a thin paper cylinder, and this cylinder flattened so as to reduce the region (inside) in which the magnetic field is intense. This gives a noninductive coil of which the electrostatic capacity is inconsiderable.

59. Moment of inertia, analogue of inductance.—The kinetic energy of a rotating wheel resides in the various moving particles of the wheel. The velocity (linear) of each particle of the wheel is proportional to the speed (angular velocity) of the wheel, and the energy of each particle is proportional to the square of the velocity, that is, to the square of the speed. Therefore, the total kinetic energy of the wheel is proportional to the square of the speed. That is,

$$W = \tfrac{1}{2}K\omega^2 \qquad (40)$$

in which W is the total energy of a wheel rotating at angular velocity ω and $(\frac{1}{2}K)$ is the proportionality factor. The quantity K is called the *moment of inertia* of the wheel.

60. Electromotive force required to make a current in a coil change.—A current once established in a coil of zero resistance would continue to flow without the help of an e. m. f. to maintain it, just as a wheel continues to turn when once started, provided there is no resistance to the motion of the wheel. To increase the speed of the wheel a torque must act upon it in the direction of its rotation, and to increase the current in the coil an e. m. f. must act on the coil in the direction of the current.

When an e. m. f., e (over and above the e. m. f. required to overcome the resistance of the coil), acts upon a coil the current is made to increase at a definite *rate*, $\frac{di}{dt}$, such that

$$e = L\frac{di}{dt}. \qquad (41)$$

Proof of equation (41). Multiplying both members of this equation by the current i we have $ei = Li\frac{di}{dt}$. Now ei is the rate, $\frac{dW}{dt}$, at which work is done on the coil in addition to work used to overcome resistance, and this must be equal to the rate at which the kinetic energy of the current in the coil increases. Differentiatin equation (39) we have $\frac{dW}{dt} = Li\frac{di}{dt}$. Therefore, equation (41) is proven.

Torque required to make the speed of a wheel increase. When a torque, T (over and above the torque required to overcome the frictional resistance), acts upon a wheel, then the angular velocity, ω, of the wheel is made to increase at a definite *rate*, $\frac{d\omega}{dt}$, such that

$$T = K\frac{d\omega}{dt}. \tag{42}$$

Proof of equation (42). Multiplying both members of this equation by the angular velocity, ω, of the wheel we have $T\omega = K\omega\frac{d\omega}{dt}$. Now $T\omega$ is the rate $\frac{dW}{dt}$ at which work is done on the wheel and this must be equal to the rate at which the kinetic energy of the wheel increases. Differentiating equation (40) we have $\frac{dW}{dt} = K\omega\frac{d\omega}{dt}$. Therefore, equation (42) is proven.

61. Magnetic flux through a coil due to a current in the coil.—In dealing with coils it is usual to speak of the *magnetic flux through the coil* as the product of the flux through the opening of the coil, or flux through a mean turn, multiplied by the number of turns. That is

$$N = ZN' \tag{43}$$

in which N' is the flux through the opening of a coil (through a mean turn), Z is the number of turns of wire in the coil and N is what is called the *flux through the coil.*

When a current i flows in a coil of wire the magnetic flux, N, through the coil is proportional to i so that

$$N = Li \tag{44}$$ *

in which L is the proportionality factor. It is shown in the next article that this L is the same as the inductance of the coil as defined in article 57.

Remark: When the current i in a coil changes at a rate $\frac{di}{dt}$

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* In this equation L and i must be expressed in c. g. s. units because the unit of flux corresponding to the ampère-henry *is not much used.*

the magnetic flux through the coil due to the current changes also so that

$$\frac{dN}{dt} = L\frac{di}{dt}$$

according to equation (44), and this changing flux induces an e. m. f. in the coil such that $e = -\frac{dN}{dt}$ or

$$e = -L\frac{di}{dt}. \tag{45a}$$

This induced e. m. f. is equal and opposite to the outside e. m. f. which, according to equation (41), must act upon the coil to make the current change. The relation between these two e. m. f's. viz., $+L\frac{di}{dt}$ and $-L\frac{di}{dt}$ is discussed in the next article.

62. Self-induced e. m. f. *Reaction of a changing current.*—When one pushes on a wheel causing its speed to increase, the wheel reacts and pushes back against the hand. This reacting torque is equal and opposite to the acting torque $K\frac{d\omega}{dt}$ [equation (42)] which is causing the increase of speed. Thus when the speed of the wheel is increasing the reacting torque is in a direction opposite to the speed and when the speed is decreasing the reacting torque is in the same direction as the speed.

Similarly when an e. m. f. acts upon a circuit * causing the current to increase, the increasing current reacts. This reacting e. m. f. is equal and opposite to the acting e. m. f. $L\frac{di}{dt}$ [equation (41)] which is causing the increase of current. This reacting e. m. f. is called a *self-induced* e. m. f. This self-induced e. m. f. is therefore

$$e = -L\frac{di}{dt}. \tag{45b}$$

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* Supposed to have zero resistance for the sake of simplicity of statement.

When a current is increasing in a coil the reaction (self-induced e. m. f.) is in a direction opposite to the current. When the current is decreasing the self-induced e. m. f. is in the same direction as the current.

Remark: The identity of equations (45)a and (45)b is a proof of equation (44).

63. Proposition.—*The inductance of a coil wound on a given spool is proportional to the square of the number of turns, Z, of wire.* For example, a given spool wound with No. 16 wire has 500 turns and an inductance of, say, 0.025 henry; the same spool wound with No. 28 wire would have about ten times as many turns and its inductance would be about 100 times as great or 2.5 henrys.

Proof: To double the number of turns on a given spool would everywhere double the field intensity for the same current, and therefore the energy of the field would everywhere be quadrupled for given current so that the inductance would be quadrupled according to equation (39).

64. Proposition.—*The inductance of a coil of given shape is proportional to its linear dimensions, the number of turns of wire being unchanged.* For example, a given coil has an inductance of 0.022 henry; and a coil three times as large in length, diameter, etc., has an inductance of 0.066 henry.

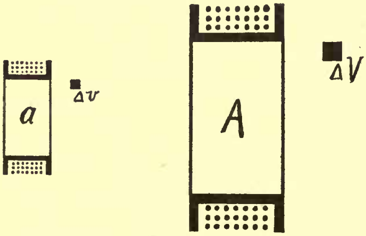

FIG. 33.

Proof: Consider a coil *a* and a similar coil *A twice* as large. See Fig. 33. Consider corresponding volume elements Δv and ΔV of the magnetic fields due to *a* and to *A* respectively. Then $\Delta V = 2^3 . \Delta v$. Further the magnetic field intensity at Δv is twice as great for given current as at ΔV according to equation (201) (text-book) so that the kinetic energy per unit volume is four times as great at Δv as at ΔV. Therefore the amount of energy in the element ΔV is twice as great as in Δv, or the total energy of the coil *A* is twice as great as the energy of the coil *a* for given current. Consequently the inductance of *A* is twice as great as the inductance of *a* according to equation (39).

65. Calculation of inductance in terms of magnetic flux per unit current.—According to equation (44) the inductance of a coil is equal to the quotient $\frac{N}{i}$ where N is the magnetic flux through the coil* due to current i in the coil. There are important cases in which the flux through a coil due to given current may be easily calculated and therefore the inductance of such a coil is easily determined.

Long solenoid. Consider a long cylindrical coil of wire of mean radius r and having n turns of wire per unit length. The field intensity in this coil is $f = 4\pi ni$ according to equation (276), text-book, and the area of the opening of the coil is πr^2 so that the flux through the opening is $4\pi^2 r^2 ni (= N')$. If we consider length l of the coil having ln turns, the flux through this portion of the coil is $4\pi^2 r^2 n^2 li (= ln.N')$ which divided by i gives the inductance of the coil, so that

$$L = 4\pi^2 r^2 n^2 l. \tag{46}$$

This equation is strictly true for very long coils only. For short coils the value of L given by this equation is too great; however, this equation is very useful in enabling one to calculate easily the approximate inductance of even a short coil.

Coil wound on an iron core. Consider a coil of Z turns of wire wound on an iron ring of peripheral length l and of sectional area q. (See Fig. 34.) The magnetomotive force of this coil, with current i, is $4\pi Zi$ according to equation (275), the flux through the rod is

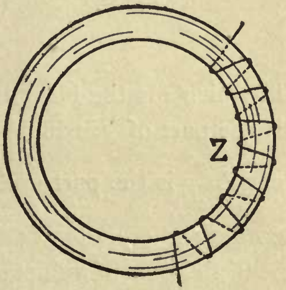

FIG. 34.

$$\frac{4\pi Zi}{\frac{1}{\mu}\frac{l}{q}} (= N')$$

---

* That is the flux through a mean turn multiplied by the number of turns of wire.

according to equation (286), and the flux through the coil is ZN' or

$$N = \frac{4\pi Z^2 i}{\frac{1}{\mu}\frac{l}{q}}.$$

Therefore, the inductance of the coil is

$$L = \frac{4\pi\mu q Z^2}{l}. \qquad (47)$$

Remark: The permeability μ of iron decreases with increasing magnetizing force. Therefore, the inductance of a coil wound on an iron core is not a definite constant as in case of a coil without an iron core.

66. Growth and decay of current in an inductive circuit.—Consider a circuit of inductance L and resistance R to which a constant e. m. f., E, is connected. A perceptible interval of time elapses before the full value, $\frac{E}{R}$, of the current is established in the circuit. During this time the current is growing, and the following equation is satisfied at each instant:

$$E = Ri + L\frac{di}{dt} \qquad (48)$$

in which i is the instantaneous value of the growing current; Ri is the part of E which overcomes the resistance of the circuit, and $L\frac{di}{dt}$ is the part of E which causes the current to increase, according to equation (41).

If a circuit of inductance L and resistance R with a given current is left to itself without any e. m. f. to maintain the current the current dies away, or decays, and the e. m. f., Ri, which, at each instant, overcomes the resistance is the self-induced e. m. f. $-L\frac{di}{dt}$, so that at each instant $Ri = -L\frac{di}{dt}$ or

$$0 = Ri + L\frac{di}{dt}. \qquad (49)$$

Example. An e. m. f. of 110 volts acts on a coil of which the inductance is 0.04 henry and the resistance is 3 ohms. At the instant that the e. m. f. begins to act the actual current i in the coil is zero and the whole of the e. m. f. acts to increase the current so that 110 volts $=$ 0.04 henry $\times \frac{di}{dt}$, or $\frac{di}{dt} =$ 2750 ampères per second. When the growing current has reached a value of 30 ampères, Ri is equal to 90 volts and the remainder of the 110 volts acts to cause the current to increase, that is 30 volts $=$ 0.04 henry $\times \frac{di}{dt}$, or $\frac{di}{dt} =$ 750 ampères per second.

If a current is established in this coil and the coil left to itself, short-circuited, without any e. m. f. to maintain the current; then, as the decaying current reaches a value of, say, 30 ampères, the e. m. f. Ri is 90 volts and this e. m. f. is equal to $-L\frac{di}{dt}$ so that $\frac{di}{dt}$ is $-$ 2250 ampères per second.

The algebraic expression for the growing current in an inductive current is

$$i = \frac{E}{R} - \frac{E}{R} e^{-\frac{R}{L}\cdot t} \qquad (50)$$

in which e is the Naperian base, and i is the value of the current t seconds after the e. m. f. E is connected to the circuit. This equation is true, for in the first place $i = o$ when $t = o$; and in the second place it satisfies equation (50) as may be shown by differentiation. The ordinates of the curve, Fig. 35, show the successive values of the growing current in an inductive circuit.

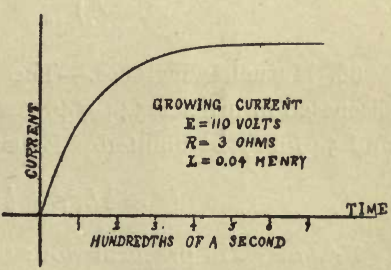

Fig. 35.

The algebraic expression for the decaying current in a short-circuited inductive circuit is

$$i = Ie^{-\frac{R}{L}\cdot t} \qquad (51)$$

in which I is the value of the decaying at the instant from which time is reckoned ($t = 0$) and i is the value of the current t seconds afterwards. The ordinates of the curve, Fig. 36, show the successive values of a decaying current in a short-circuited inductive circuit.

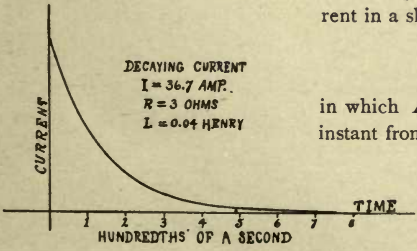

Fig. 36.

67. Circulation of charge during growth or decay of a current in an inductive circuit.—During the growth of a currrent in an inductive circuit which is connected to a constant e. m. f., the current i is less than its final value $I\left(=\frac{E}{R}\right)$ so that the quantity of electric charge which has passed through the circuit by the time the current has reached its final value is less than it would have been had the current started with its full final value. This *deficiency* of charge is an important consideration in some of the methods for measuring inductance; it is

$$Q=\frac{LI}{R} \tag{52}$$

and it is equal to the charge which passes through the circuit while a current of initial value I is decaying to zero. In this equation L is the inductance and R the resistance of the circuit.

Proof of equation (52). The amount by which the actual current falls short of $I\left(=\frac{E}{R}\right)$ at time t is, by equation (50), equal to $Ie^{-\frac{R}{L}\cdot t}$ and it is this deficiency in current which produces the deficiency in the charge so that

$$\Delta Q=Ie^{-\frac{R}{L}\cdot t}\Delta t$$

or

$$Q=I\int_{t=0}^{t=\infty} e^{-\frac{R}{L}\cdot t}\cdot dt=\frac{LI}{R} \qquad \text{Q. E. D.}$$

68. Mutual Inductance.—The subject of mutual inductance is discussed in articles 544–549, text-book. Of these articles 548 and 549 may be omitted. Equation (298) should read

$$W=\tfrac{1}{2}L_1 i_1^2+\tfrac{1}{2}L_{11} i_{11}^2+M i_1 i_{11}. \tag{298}$$

Remark: In practical work the conception of Mutual Inductance is not of great importance inasmuch as the inductive relation of two coils is merely a matter of the sending of magnetic flux through the one by a current in the other.

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